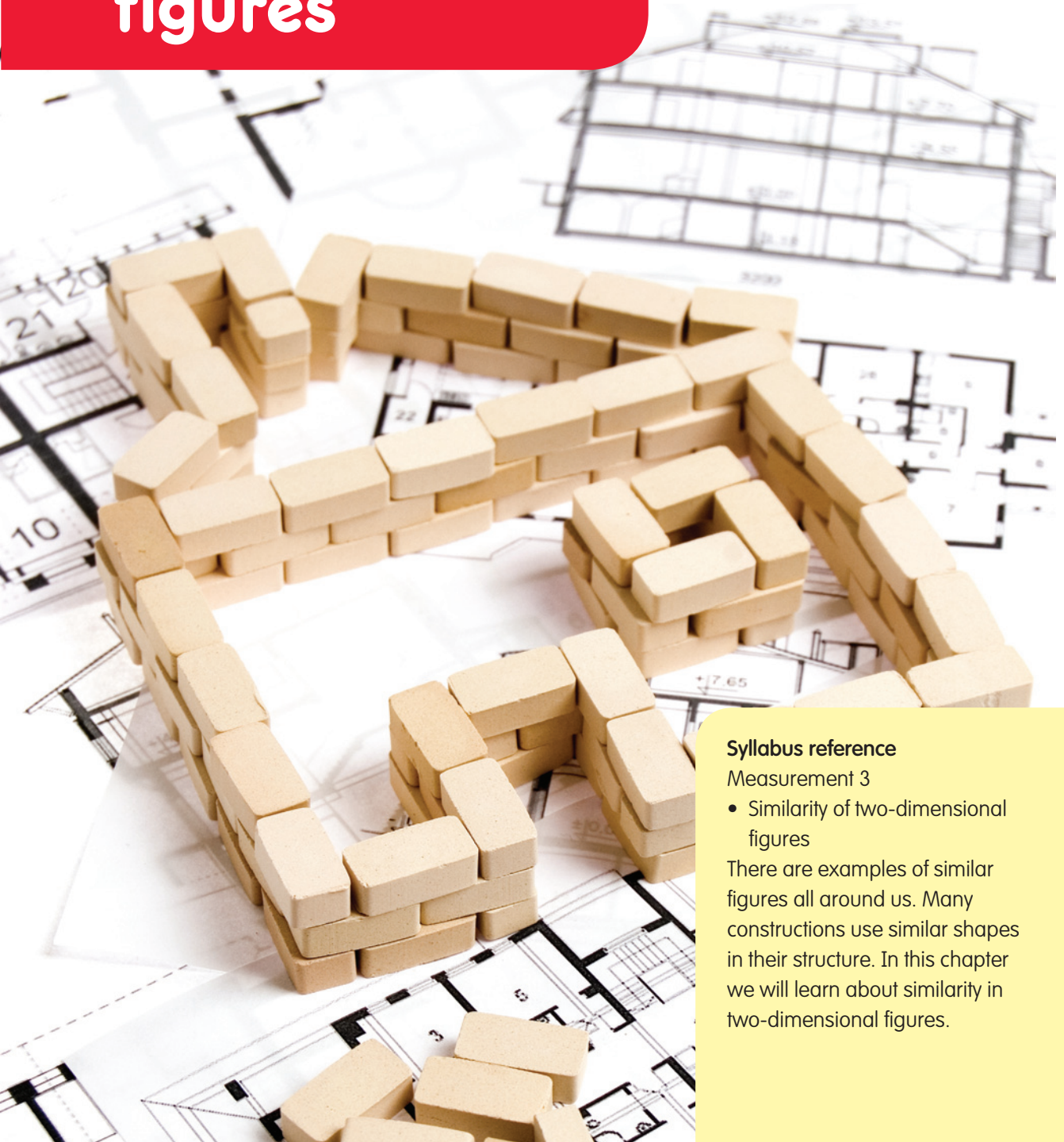


# Similarity of two-dimensional figures



## Syllabus reference

### Measurement 3

- Similarity of two-dimensional figures

There are examples of similar figures all around us. Many constructions use similar shapes in their structure. In this chapter we will learn about similarity in two-dimensional figures.

# ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillsHEET. Either click on the SkillsHEET icon next to the question on the *Maths Quest Preliminary Course* eBookPLUS or ask your teacher for a copy.

## eBookplus

### Digital doc

SkillsHEET 11.1  
doc-1596

### Simplifying ratios

## Simplifying ratios

1 Simplify each of the following ratios.

a  $15 : 12$

b  $\$56 : \$49$

c  $0.8 : 1.25$

d  $40 \text{ cm} : 2 \text{ m}$

## eBookplus

### Digital doc

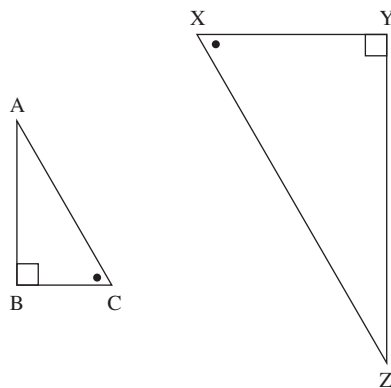
SkillsHEET 11.2  
doc-1597

### Corresponding sides of congruent and similar triangles

2 Consider the similar triangles drawn at right.

a Which side in  $\triangle XYZ$  corresponds to  $BC$ ?

b Which angle in  $\triangle ABC$  corresponds to  $\angle XZY$ ?



## eBookplus

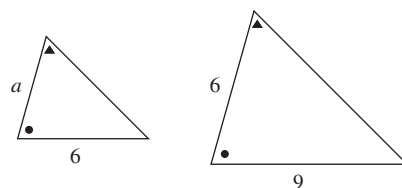
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### Similar triangles

## Similar triangles

3 Find the value of the pronumeral in the figure at right.



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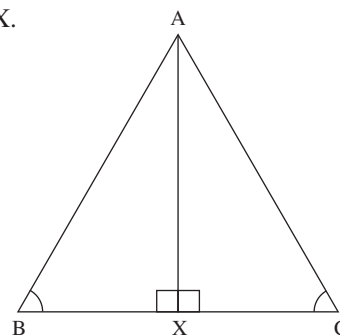
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SkillsHEET 11.4  
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### Using tests to prove similar triangles

## Using tests to prove similar triangles

4 Using the figure at right, prove that  $\triangle ABX$  is similar to  $\triangle ACX$ .



## eBookplus

### Digital doc

SkillsHEET 11.5  
doc-1600

### Finding the scale factor

## Finding the scale factor

5 Find the scale factor given that:

a  $AB = 6 \text{ cm}$  and  $A'B' = 18 \text{ cm}$

b  $AB = 30 \text{ cm}$  and  $A'B' = 27 \text{ mm}$ .

# 11A Similar figures and scale factors

Have you ever read a road map or looked at plans for a house? The map or the plan is a scaled down version of the roads or house. When two objects are identical, except one is a **reduction** or an enlargement of the other, the objects are said to be *similar*. Maps and plans are practical examples of *similarity*.

Maps and plans both use a scale. The scale tells us how many times larger an object is in reality compared to the plan. For example, a house plan may use a scale of 1 : 100. This means that if a wall is 1 cm long on the plan, it is 100 cm (or 1 m) in reality. All the angles shown on the plan are the same as in reality. If two walls meet at right angles on the plan, they meet at right angles in reality.

**Similar figures** are in proportion and have the same shape. That is, each pair of corresponding sides are in the same ratio and each pair of corresponding angles are equal.

To show that two triangles are similar, we can show that either of the above conditions is true. The symbol for similarity is three vertical lines ( $\parallel\parallel\parallel$ ).

For triangles, if two pairs of corresponding sides are in the same ratio and the angles they include are equal then they are similar.

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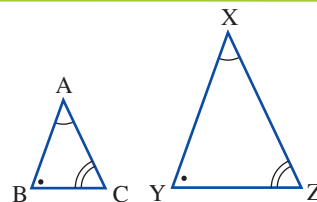
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**Similarity**

## WORKED EXAMPLE 1

In the figure at right, show that  $\triangle ABC \parallel\parallel\parallel \triangle XYZ$ .



### THINK

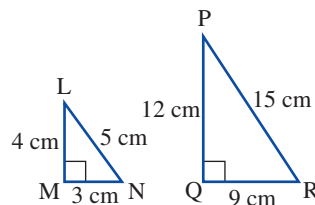
- 1  $\angle BAC$  and  $\angle YXZ$  are equal.
- 2  $\angle ACB$  and  $\angle XZY$  are equal.
- 3  $\angle ABC$  and  $\angle XYZ$  are equal.
- 4 Make a conclusion.

### WRITE

$$\begin{aligned}\angle BAC &= \angle YXZ \\ \angle ACB &= \angle XZY \\ \angle ABC &= \angle XYZ \\ \triangle ABC &\parallel\parallel\parallel \triangle XYZ \text{ (3 pairs of equal angles)}\end{aligned}$$

## WORKED EXAMPLE 2

Show that the triangles LMN and PQR are similar.



### THINK

- 1 Simplify the ratio LM : PQ.
- 2 Simplify the ratio MN : QR.
- 3 Simplify the ratio LN : PR.
- 4 Make a conclusion.

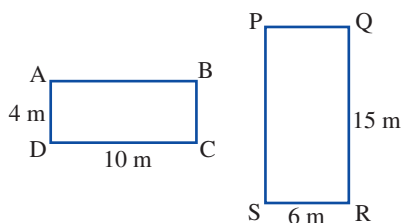
### WRITE

$$\begin{aligned}\text{LM : PQ} &= 4 : 12 \\ &= 1 : 3 \\ \text{MN : QR} &= 3 : 9 \\ &= 1 : 3 \\ \text{LN : PR} &= 5 : 15 \\ &= 1 : 3 \\ \triangle \text{LMN} &\parallel\parallel \triangle \text{PQR} \text{ (3 pairs of sides in equal ratio)}\end{aligned}$$

To determine if other figures are similar, we need to examine the ratio of sides.

### WORKED EXAMPLE 3

Determine if the rectangles ABCD and PQRS are similar.



#### THINK

- 1 Simplify the ratio of corresponding sides AD and RS.
- 2 Simplify the ratio of corresponding sides CD and QR.
- 3 Make a conclusion.

#### WRITE

$$\begin{aligned} AD:RS &= 4:6 \\ &= 2:3 \end{aligned}$$

$$\begin{aligned} CD:QR &= 10:15 \\ &= 2:3 \end{aligned}$$

The rectangles are similar as their corresponding sides are in equal ratio.

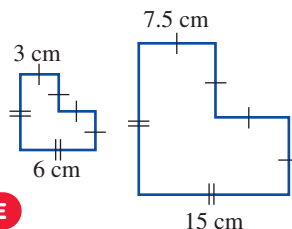
When we examine similar figures we can state the ratio of sides between the two figures. The number by which we multiply measurements on the first figure to get the measurements on the second figure is called the **scale factor**.

The scale factor is calculated by replacing the first part of the ratio of sides with one. The second part of the ratio is then calculated and is the scale factor.

### WORKED EXAMPLE 4

The two figures at right are similar.

- a What is the ratio of their sides?
- b What is the scale factor?



#### THINK

- a
  - 1 The 6 cm side corresponds to the 15 cm side. Write this as a ratio and simplify.
  - 2 The 3 cm side corresponds to the 7.5 cm side. Check that this simplifies to the same ratio.
  - 3 Make a conclusion.
- b
  - 1 The scale factor is written by comparing one unit on the first figure with the second.
  - 2 Make a conclusion.

#### WRITE

$$6:15 = 2:5$$

$$\begin{aligned} 3:7.5 &= 30:75 \\ &= 2:5 \end{aligned}$$

The similar figures are in the ratio 2:5.

$$2:5 = 1:2\frac{1}{2}$$

The scale factor is  $2\frac{1}{2}$ .

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Tutorial  
int-2329

Worked example 4

A special case of similarity occurs when the scale factor is 1. These shapes are identical and are called **congruent figures**.

## REMEMBER

1. Similar figures have all pairs of corresponding angles equal and all pairs of corresponding sides in equal ratio.
2. To prove that triangles are similar, we need to prove only that either the corresponding angles are equal or that the corresponding sides are in equal ratio.
3. The ratio of the corresponding sides in similar figures can be used to calculate the scale factor.
4. If the scale factor is 1 then the figures are congruent.

## EXERCISE

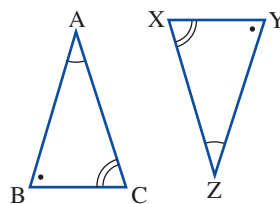
### 11A

## Similar figures and scale factors

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**Simplifying ratios**

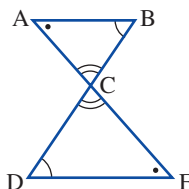
- 1 **WE1** Prove that  $\triangle ABC \parallel \triangle ZYX$ .



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**Digital doc**  
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**Corresponding sides of congruent and similar triangles**

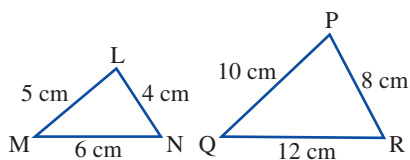
- 2 Prove that  $\triangle ABC \parallel \triangle EDC$ .



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**Similar triangles**

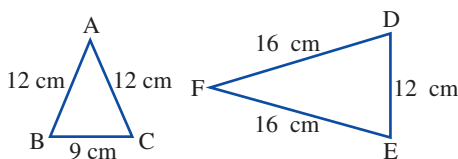
- 3 **WE2** Prove that  $\triangle LMN \parallel \triangle PQR$ .



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**Using tests to prove similar triangles**

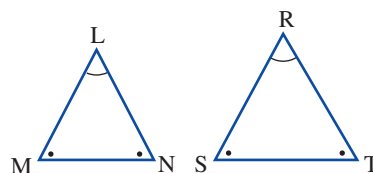
- 4 Prove that  $\triangle ABC \parallel \triangle FED$ .



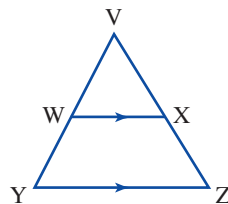
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**Finding the scale factor**

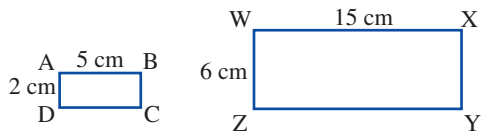
- 5 Prove that  $\triangle LMN \parallel \triangle RST$ .



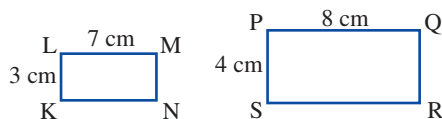
- 6 Prove that  $\triangle VWX \parallel \triangle VYZ$ .



- 7 **WE3** Determine if the rectangles ABCD and WXYZ are similar.



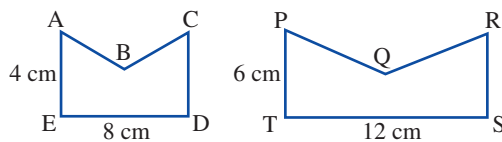
- 8 Determine if the rectangles KLMN and PQRS are similar.



- 9 **WE4** The figures at right are similar.

a What is the ratio of sides?

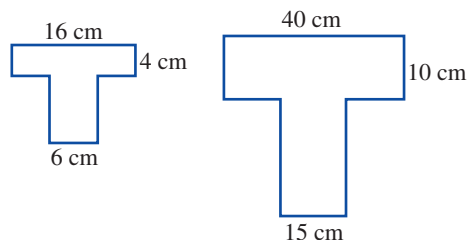
b What is the scale factor?



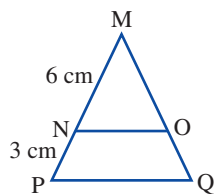
- 10 The figures at right are similar.

a What is the ratio of sides?

b What is the scale factor?



- 11 In the figure below,  $\triangle MNO \parallel \triangle MPQ$ . Calculate the ratio of sides.



- 12 On a set of house plans, a measurement of 5 cm represents a wall which is 10 m long. Calculate the scale factor.

- 13 On a map, a distance of 3 cm represents an actual distance of 60 km. Calculate the scale factor.



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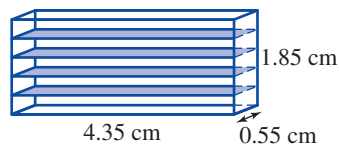
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## Further development

- 14** Scale factors can be given as comparative distances or as ratios. Convert each of these scale factors, given as comparative distances, to ratios.
- a** 1 mm to 1 m                      **b** 2 cm to 16 m                      **c** 4 cm to 25 m  
**d** 40 cm to 1 m                      **e** 20 cm to 10 cm                      **f** 375 mm to 1 m
- 15** Write each of the ratios in question **14** as a rule with the real life (*RL*) as the subject of a formula in terms of scale length (*SL*). For example part (a)  $RL = 1000 \times SL$ .
- 16** A scale plan for the construction of a television unit is drawn. The scale of the plan is 3 cm to 20 cm. The height of the unit on the plan is 15 cm.
- a** What will be the real life height of the unit?  
**b** The real life width of the unit is to be 125 cm. What will be the scale width of the unit on the plan?
- 17** In a furniture catalogue you decide to buy a bookcase that is 74 cm high. Below is a diagram of the bookcase with the measurements as shown on a scale diagram.



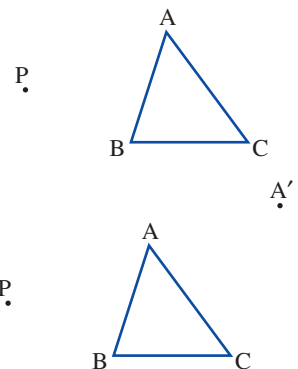
- a** What is the scale used?  
**b** Find the length and width of the bookcase.
- 18** The photo at right is of a 30 metre tall tree.
- a** The tree is 5 cm tall in the photo. Find the scale factor as a ratio.  
**b** The diameter of the tree is 1.2 m. Find the diameter of the tree in the photo.
- 19** State whether each of the following statements are true or false.
- a** All squares are similar.  
**b** All rectangles are similar.  
**c** All triangles are similar.  
**d** All equilateral triangles are similar.  
**e** All circles are similar.



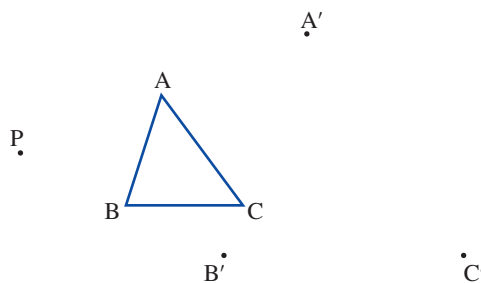
### INVESTIGATE: Enlarging a figure

We can draw similar figures using an **enlargement** factor. We will enlarge the triangle below by a scale factor of 2.

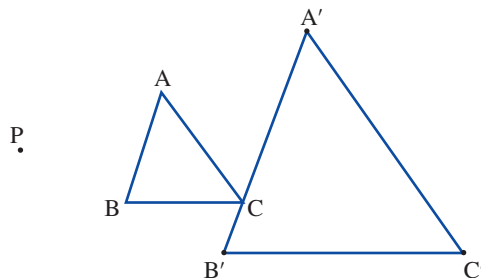
- 1** Mark a point, P, external to the figure. This point is called the centre of enlargement.
- 2** Measure the distance from P to the vertex, A. Mark a point twice this distance away in a straight line. Label this point A'.



- 3 Repeat step 2 for the vertices B and C.



- 4 Join the points A', B' and C'.



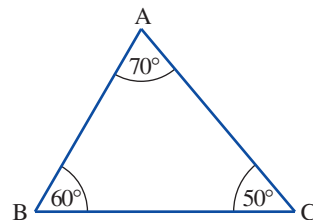
### INVESTIGATE: Investigating scale factors

- 1 Draw a figure on clear plastic so that it can be placed on an overhead projector.
- 2 Place the overhead projector 2 m from the screen and focus the image. Measure the lengths on the image and state the scale factor.
- 3 Repeat step 2, placing the overhead projector 3 m, 4 m and 5 m from the screen.
- 4 Determine if there is a relationship between the scale factors and the distance from the projector to the screen.

### INVESTIGATE: Similar triangles

Two triangles are similar if they have the same shape but not necessarily the same size. One is an enlargement or reduction of the other. This means that the corresponding angles of the triangles have to be equal (to make them the same shape) and the ratio of their corresponding sides must be constant (making one smaller or larger than the other). As with congruent triangles, we do not need to know *all* the information about the three sides and three angles to determine if a pair of triangles is similar. Certain minimum information is sufficient. Let us investigate.

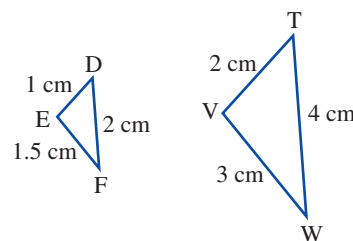
- 1 Draw the  $\triangle ABC$  shown (it is not drawn to scale). Draw  $\triangle XYZ$ , larger than  $\triangle ABC$  with  $\angle X = \angle A$ ,  $\angle Y = \angle B$  and  $\angle Z = \angle C$ . Measure the lengths of the sides of the two triangles. Determine the ratios of the lengths of the corresponding sides  $\frac{XY}{AB}$ ,  $\frac{YZ}{BC}$  and  $\frac{ZX}{CA}$ . Are these ratios constant (within the limits



of the accuracy of the constructions)? Does it appear that  $\triangle XYZ$  is a true enlargement of  $\triangle ABC$ ? Repeat the process, drawing  $\triangle XYZ$  smaller than  $\triangle ABC$ . Is  $\triangle XYZ$  similar to  $\triangle ABC$ ?

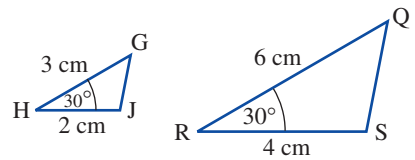
- 2 Construct the two triangles shown where  $\triangle TVW$  is twice the size of  $\triangle DEF$ .

The ratio of their corresponding sides is constant as  $\frac{TV}{DE} = \frac{VW}{EF} = \frac{WT}{FD} = 2$ . Measure their corresponding angles. Are the two triangles similar?

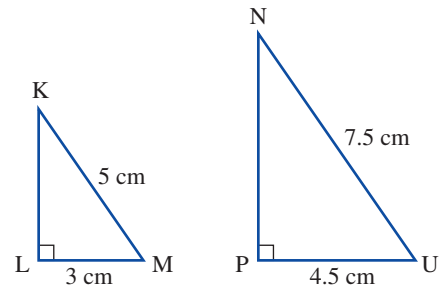




- 3 Construct  $\triangle GHJ$  and  $\triangle QRS$  to the measurements shown at right. Find the ratio of their corresponding sides (as in part 1) and measure all angles. What do you conclude?



- 4 Draw the right-angled triangles KLM and NPU to the dimensions given. Again, find the ratio of their corresponding sides (as in part 1) and measure all angles. What do you conclude?



- 5 Summarise the results of your investigation. What are the minimum requirements to ensure the similarity of two triangles?

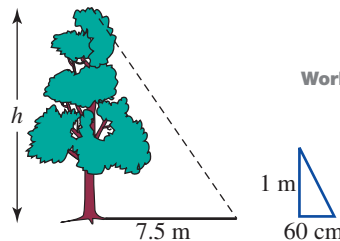
## 11B Solving problems using similar figures

We can use similar figures to solve many problems. By setting up similar triangles we can calculate measurements of objects such as trees, which we are unable to physically measure. Another example is house plans. In this case, the ratio of sides becomes the scale of the plan making it similar to the house itself.

Consider the case where we want to measure the height of a tree too tall for us to physically measure. Using shadows we can create two similar triangles.

### WORKED EXAMPLE 5

A tree casts a shadow 7.5 m long. At the same time a 1-metre ruler casts a shadow 60 cm long. Calculate the height of the tree.



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Worked example 5

#### THINK

- The triangles are similar because all three angles are equal.
- Write the shadow lengths as a ratio and simplify.
- Write a proportion and solve to find the height of the tree.

#### WRITE

$$7.5 \text{ m} : 60 \text{ cm} = 750 : 60 \\ = 25 : 2$$

$$\frac{h}{1} = \frac{25}{2} \\ h = 12.5 \text{ m}$$

We use a similar method when reading maps or plans. The map is a similar figure to the place being mapped. We use the scale given on the map to calculate the distance between two places.

### WORKED EXAMPLE 6

The scale on a road map is given as  $1 \text{ cm} = 5 \text{ km}$ . Jodie uses her ruler and finds the distance between the towns Huxley and Brownville is  $6.2 \text{ cm}$ . Calculate the distance between these two towns.

#### THINK

- 1 Multiply  $6.2 \text{ cm}$  by the given scale.
- 2 Give a written answer.

#### WRITE

$$6.2 \text{ cm} \times 5 \text{ cm/km} = 31 \text{ km}$$

The actual distance between Huxley and Brownville is  $31 \text{ km}$ .

In the case of plans, the scale is often stated as a ratio. The method of solution is the same.

### WORKED EXAMPLE 7

The scale on a house plan is  $1 : 150$ . The front of the house measures  $8.5 \text{ cm}$  on the plan. Calculate the actual length of the front of the house.

#### THINK

- 1 Multiply the measurement by the scale.
- 2 Change the units from  $\text{cm}$  to  $\text{m}$ .
- 3 Give a written answer.

#### WRITE

$$\begin{aligned} 8.5 \text{ cm} \times 150 &= 1275 \text{ cm} \\ &= 12.75 \text{ m} \end{aligned}$$

The front of the house is  $12.75 \text{ m}$  long.

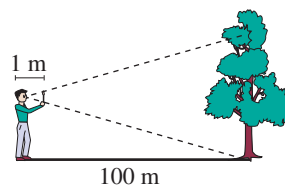
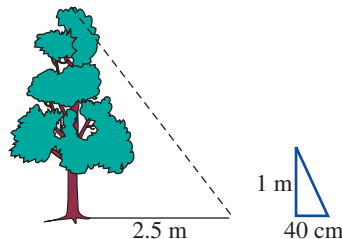
### REMEMBER

1. We can calculate measurements of objects by setting up similar triangles.
2. Maps and plans are similar to the actual object.
3. The scale is the ratio of sides on the plan to those on the object. This scale can be used to make calculations.

### EXERCISE

## 11B Solving problems using similar figures

- 1 **WES** A tree casts a shadow  $2.5 \text{ m}$  long. At the same time a  $1\text{-metre}$  ruler casts a shadow  $40 \text{ cm}$  long, as shown in the figure. Calculate the height of the tree.
- 2 A building casts a shadow  $9.5 \text{ m}$  long. At the same time a  $1\text{-metre}$  ruler casts a shadow  $25 \text{ cm}$  long.
  - a Draw a diagram to represent this situation.
  - b Calculate the height of the building.
- 3 Kerry is  $170 \text{ cm}$  tall and her shadow measures  $50 \text{ cm}$ . At the same time a flagpole casts a shadow which is  $3 \text{ m}$  long.
  - a Draw a diagram to represent this situation.
  - b What is the ratio of sides in the similar triangles formed?
  - c Calculate the height of the flagpole.
- 4 An artist holds his brush so that the top and bottom of the brush line up with the top and bottom of a tree. The brush is  $10 \text{ cm}$  long and is held  $1 \text{ m}$  away from the artist's eye. The tree is  $100 \text{ m}$  away from the artist. Calculate the height of the tree.



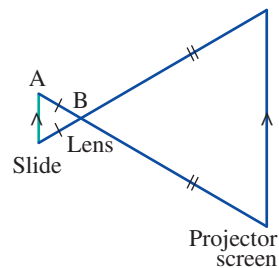
- 5 **MC** A tree casts a 6 m shadow. At the same time a 1-metre ruler casts a shadow 1.2 m long. The ratio of sides in the similar figures formed is:  
**A** 5:6                      **B** 5:1                      **C** 6:1                      **D** 10:1
- 6 **WE6** A map gives the scale as being 1 cm = 10 km. Two towns are shown as being 6 cm apart on the map. What is the actual distance between the two towns?
- 7 On a map where the scale is given as 1 cm = 4 km, calculate the actual distance where the distance on the map is measured as:  
**a** 5 cm                      **b** 9 cm                      **c** 6.5 cm  
**d** 12.8 cm                      **e** 0.9 cm                      **f** 4 mm.
- 8 On a map where the scale is given as 1 cm = 5 km, what should the distance on the map be when the actual distance is:  
**a** 20 km?                      **b** 45 km?  
**c** 22 km?                      **d** 340 km?  
**e** 8 km?                      **f** 37.5 km?
- 9 **MC** The scale on a map is given as 1 cm = 4 km. If the distance between two points on the map is 3.8 cm, then the actual distance between these two points is:  
**A** 15.2 cm                      **B** 3.8 km  
**C** 4 km                      **D** 15.2 km
- 10 **MC** On a map the scale is given as 1 cm = 5 km. The distance from Freewell to Taleton is 64 km. How far apart should they be drawn on the map?  
**A** 0.078 125 cm                      **B** 12.8 cm                      **C** 320 cm                      **D** 12.8 km
- 11 **WE7** The scale on a set of house plans is given as 1 : 500. Calculate the length of the house frontage if it is shown as 4 cm on the plans.
- 12 A set of house plans is to be drawn using a scale of 1 : 400. The side of the house is to be 16 m long. Calculate the length that this should be drawn on the plans.



### Further development

- 13 Solve each of the following by drawing a pair of similar triangles.
- A low bridge casts a shadow that is 1.44 metres long at the opening. A handyman's truck is 2.5 metres high. To determine if the truck will pass under the bridge the handyman gets out of the truck and finds that a 1.5 m rod casts a 90 cm shadow. Will the truck pass under the bridge? Explain your answer.
  - Steve Hooker is Australia's pole vault gold medallist from the Beijing Olympics. Steve's 1.8 m pole casts a 96 cm shadow on the ground at the same time as the bar he is about to vault casts a 2.4 m shadow. How high is the bar on the pole vault?
  - The fire brigade need to rescue people from atop a building. Their 15 metre ladder casts a 20 metre shadow while the building casts a 24 metre shadow. What length ladder is needed?
- 14 Andrew is 1.8 m metres tall and plays basketball. He casts a 60 cm shadow on the court at the same time as the basketball pole casts a 1 metre shadow. If the basketball ring is 45 cm below the top of the pole, how high must Andrew reach to slam dunk the basketball?
- 15 A triangle with sides 18 cm, 36 cm and 48 cm is similar to a triangle that has a measure of 8 cm down the longest side.
- What is the scale factor?
  - What is the perimeter of the smaller triangle?

- 16** A data projector is to be set up to project onto a screen that is 1.5 metres high. The image is projected from a point behind the lens such that  $AB = 10$  cm and the image height is 5 cm. What is the maximum distance that the projector can be placed from the screen?



- 17** A rectangle has dimensions 8 cm by 12 cm.
- What is the area of the rectangle?
  - A similar rectangle is drawn with scale factor 2 : 5. What will be the dimensions of the larger rectangle?
  - Find the area of the larger rectangle.
  - What is the ratio of the two areas?
  - What do you notice when you compare your answer to part (d) to the scale factor?

#### INVESTIGATE: Scale drawing of the classroom

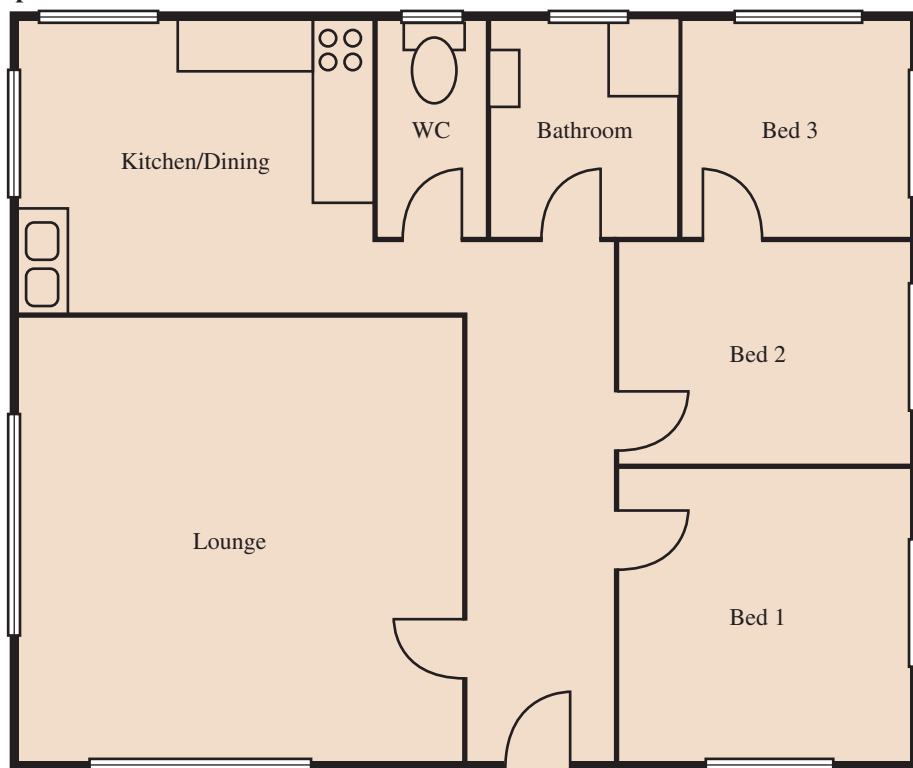
Draw a scale diagram showing the floor plan of the classroom you are now in. On your diagram show the location of all desks, cupboards, the blackboards and any other features of the room.

## 11C House plans

House plans are a very common application of similar figures. As we saw in the previous section, plans are drawn using a ratio as the scale factor. Measurement enables us to calculate all dimensions within the house. Corresponding angles on similar figures are equal and so the angles on the plans will be the same as the angles in reality.

### WORKED EXAMPLE 8

Below is a plan for a house.



Scale 1:100

- Calculate the dimensions of the house.
- Calculate the area of the lounge room.

**THINK**

- a
- 1 Measure the length and width of the house on the plan.
  - 2 Multiply each of these measurements by 100.
  - 3 Write your answer.
- b
- 1 Measure the length and width of the lounge room on the plan.
  - 2 Multiply each of these measurements by 100.
  - 3 Calculate the area of the lounge room.
  - 4 Write your answer.

**WRITE**

- a
- Length of house on plan = 12 cm  
 Width of house on plan = 10 cm  
 Actual length of house =  $12 \text{ cm} \times 100$   
 $= 1200 \text{ cm}$   
 $= 12 \text{ m}$   
 Actual width =  $10 \text{ cm} \times 100$   
 $= 1000 \text{ cm}$   
 $= 10 \text{ m}$
- The dimensions of the house are 12 m by 10 m.
- b
- Length of lounge room on plan = 6 cm  
 Width of lounge room on plan = 6 cm  
 Actual length of lounge room =  $6 \text{ cm} \times 100$   
 $= 600 \text{ cm}$   
 $= 6 \text{ m}$   
 Actual width of lounge room is also 6 m.  
 $A = 6^2$   
 $= 36 \text{ m}^2$   
 The area of the lounge room is  $36 \text{ m}^2$ .

House plans are also drawn with a view of what the house will look like from the outside. These diagrams are called **elevations**. For example, the front elevation is what the house will look like from the front. Elevations are also drawn using a scale.

**WORKED EXAMPLE 9**

The diagram below shows the front elevation of a house.

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int-2331

**Worked****example 9**

- a Calculate the height of the eaves on the lower side of the house.
- b Measure the angle of the pitch of the roof.

**THINK**

- a
- 1 Measure the height on the plan for the lower side of the house.

**WRITE**

- a
- Height on the plan = 3.5 cm



- 2 Multiply the plan measurement by 100.

$$\begin{aligned}\text{Actual height} &= 3.5 \text{ cm} \times 100 \\ &= 350 \text{ cm} \\ &= 3.5 \text{ m}\end{aligned}$$

- 3 Write your answer.

The height of the eaves is 3.5 m.

- b 1 Measure the angle that the slope of the roof makes with the horizontal.

b Angle to horizontal =  $45^\circ$

- 2 Write your answer.

The angle of the pitch of the roof is  $45^\circ$ .

### REMEMBER

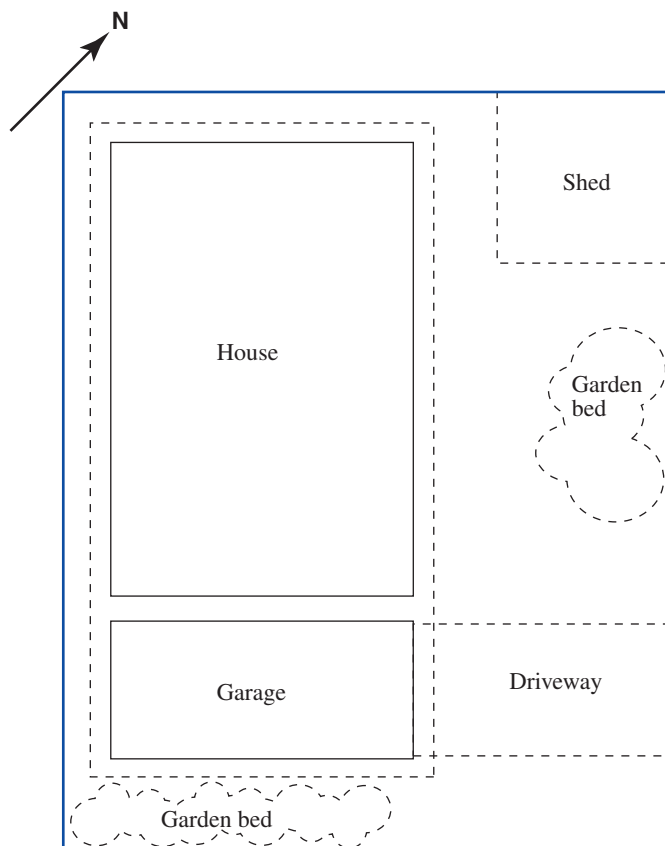
1. House plans are drawn with a ratio as the scale factor.
2. Using the scale factor, we can calculate the dimensions for the house from the plans.
3. An elevation is the view of a house from one side.
4. From an elevation, we can measure all lengths and angles and use the scale to calculate the actual measurements.

### EXERCISE

11C

## House plans

- 1 **WEB** Below is the site plan for a block of land.

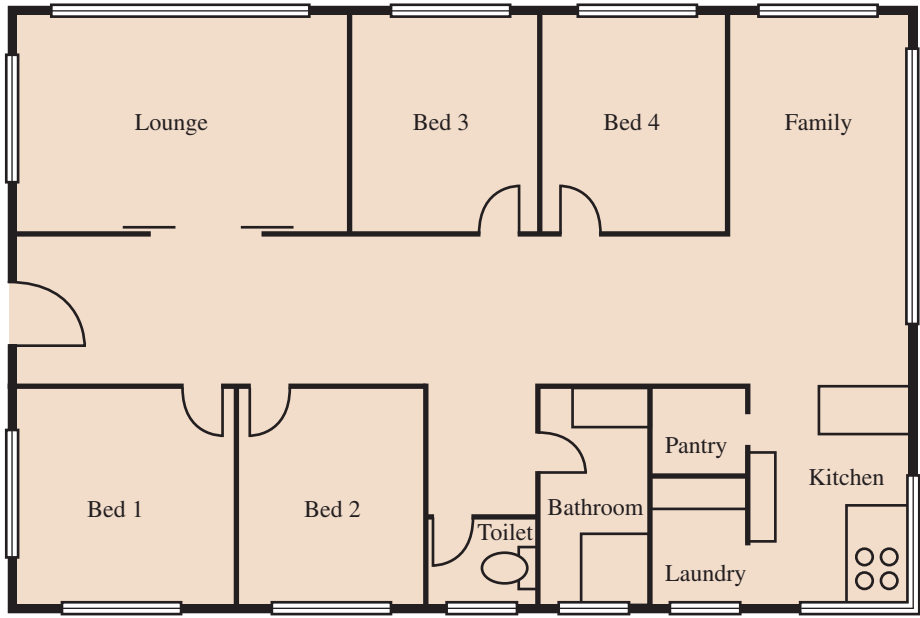


Scale 1:250

- a What are the dimensions of the block of land?  
b What are the dimensions of the house?



2 A house plan is shown below.



Scale 1:150

- a Calculate the dimensions of the house.
  - b What are the dimensions of the lounge room?
  - c Which bedroom is the largest? What are its dimensions?
- 3 **WE9** Below is the front elevation of a house, drawn to scale.

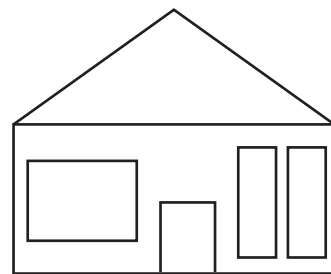


Scale 1:100

- a Calculate the height of the peak of the roof.
- b Calculate the height of the eaves.
- c Measure the angle of the pitch of the roof.

- 4 Trace the front elevation of the house at right into your book.

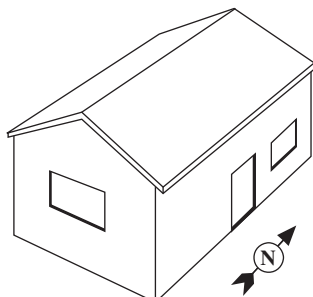
On your diagram write all lengths and angles necessary for the construction of the house.



Scale 1:200

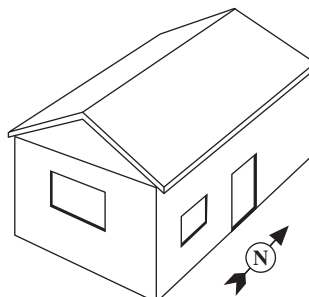
- 5 The following diagrams are representations of houses with a variety of roof types. Draw a plan of the south and east elevations of these houses. The direction of north is given.

a



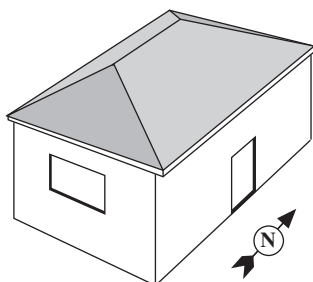
Gable roof

b



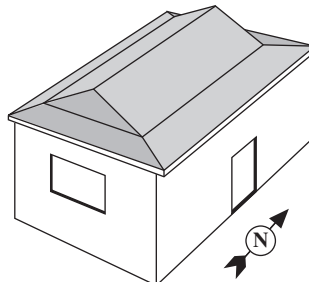
Boxed gable roof

c



Hip roof

d



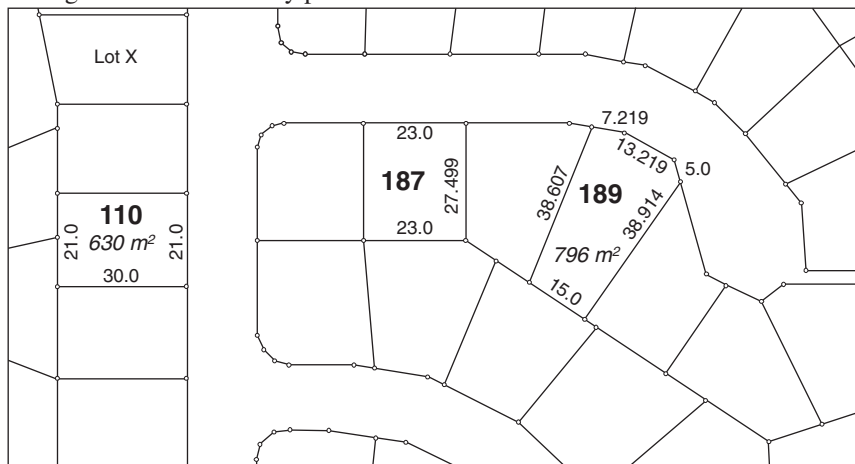
Gambrel  
(a roof combining the addition  
of small gables to a hip roof)

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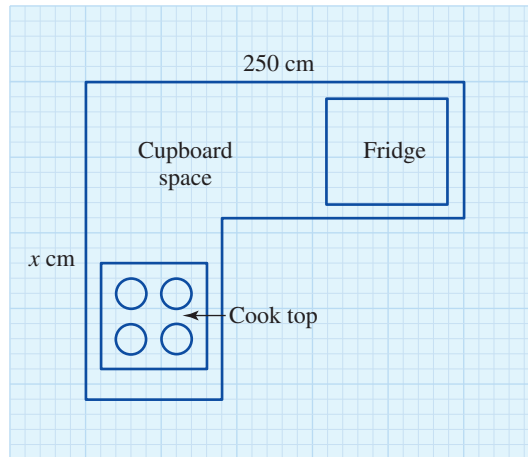
### Further development

- 6 The figure below is a survey plan of a street.





- 9 The plan below shows a simple design of a kitchen.



- What is the scale used in this plan?
- Find the dimensions of the fridge
- Find the length of the side marked  $x$ .
- Find the area of cupboard space (i.e. the space not taken up by the fridge or cook top).

#### INVESTIGATE: House plans

- Try to obtain a set of plans to a house.
  - What is the scale on the plans?
  - What are the dimensions of the house?
  - What is the total area of the house?
- What symbols are used on the house plans to indicate the following:
  - a door
  - a window
  - cupboards
  - any other significant features?
- Using a suitable scale, draw a set of plans for your house. Include a floor plan of your house and a front elevation.

# SUMMARY

## Similar figures

- Similar figures have all pairs of corresponding angles equal and corresponding sides in equal ratio.
- To show that triangles are similar, we show either that all pairs of corresponding angles are equal or that all pairs of corresponding sides are in equal ratio.
- For other figures it is necessary to show that both properties are true.

## Scale factors

- The scale factor allows us to solve problems using similar figures.
- Heights of objects, such as trees, that are not easily measured can be determined by constructing similar triangles.

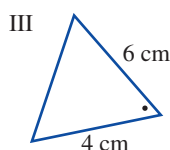
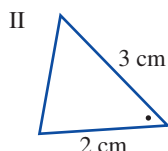
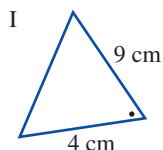
## House plans

- House plans are similar to the house being built.
- We can measure all lengths and angles on a house plan and use the scale to calculate the actual measurements.

# CHAPTER REVIEW

## MULTIPLE CHOICE

- 1 **MC** Which of the triangles below are similar?



- A I and II  
B I and III  
C II and III  
D I, II and III

- 2 **MC** Consider the statements below.

**Statement I.** All rhombuses are similar.

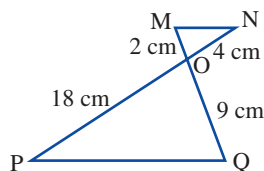
**Statement II.** All parallelograms are similar.

Which of the above statements is correct?

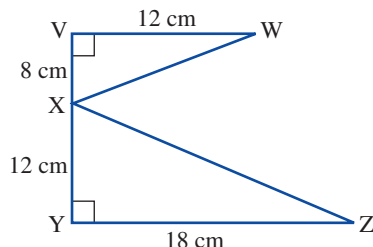
- A Statement I  
B Statement II  
C Statements I and II  
D Neither statement
- 3 **MC** On a set of house plans a room that measures  $4.5 \text{ m} \times 3.2 \text{ m}$  is shown as  $9 \text{ cm} \times 6.4 \text{ cm}$ . The scale of the plans is:
- A 1 : 2  
B 1 : 50  
C 1 : 100  
D 1 : 200

## SHORT ANSWER

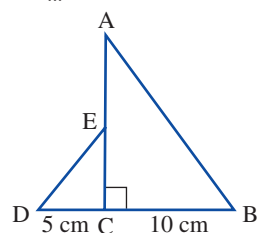
- 1 Prove that  $\triangle MNO \parallel \triangle PQO$ .



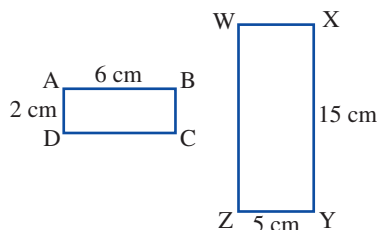
- 2 Prove that  $\triangle VWX \parallel \triangle YZX$ .



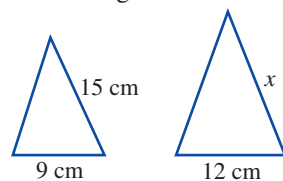
- 3 Given that E is the midpoint of the line AC, prove that  $\triangle ABC \parallel \triangle EDC$ .



- 4 ABCD and WXYZ are rectangles.  
a Prove that the two rectangles are similar.  
b State the ratio of sides in the two similar figures.



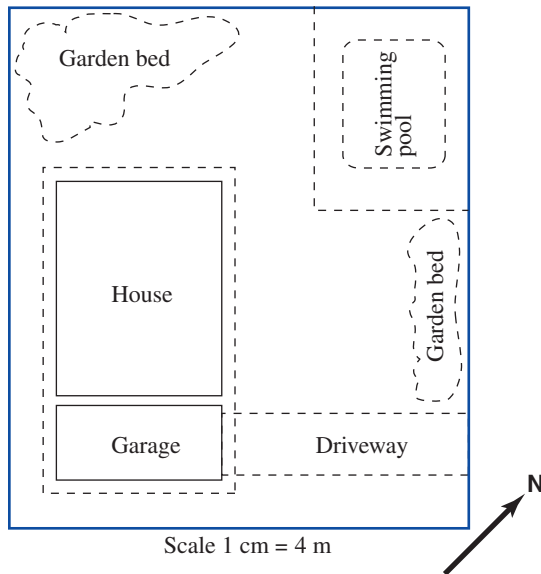
- 5 The two triangles at right are similar. Use this to find the length of the side marked x.



- 6 When a 1-metre ruler casts a shadow 75 cm long, a building casts a 15 m shadow. Calculate the height of the building.
- 7 A 10 m ladder will reach 9 m up a wall. How high up a wall will a 25 m ladder reach, if it is placed at the same angle to the ground?
- 8 A map has a scale of 1 : 40 000. Calculate the actual distance between two places if they are shown on the map as:
- a 1 cm apart    b 6 cm apart    c 8.5 cm apart.



- 9 A surveyor's plan has a scale  $1 : 800$ . The dimensions of a property are shown as  $10 \text{ cm} \times 16 \text{ cm}$  on the plan. What are the actual dimensions on the property?
- 10 The scale diagram below is a site plan for a block of land on which a house is to be built.



- a Calculate the dimensions of the block of land.
- b Calculate the dimensions of the house.
- c Calculate the area of the house in square metres.



### EXTENDED RESPONSE

- 1 Harley is  $160 \text{ cm}$  tall. He casts a shadow  $1 \text{ m}$  long.
- a Calculate the height of a tree, which at the same time casts a shadow  $4.8 \text{ m}$  long.
- b Calculate the length of the shadow that would be cast by a building that is  $20 \text{ m}$  tall.



- 2 A rectangular block of land is  $20 \text{ m}$  wide and  $32 \text{ m}$  long.
- a Using a scale of  $1 : 200$ , make a scale drawing of the block of land.
- b On the same diagram, a house is shown as  $6.2 \text{ cm}$  long and  $5.9 \text{ cm}$  wide. Calculate the actual dimensions of the house.

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Test Yourself  
doc-1603  
Chapter 11

**Are you ready?****Digital docs** (page 362)

- SkillsHEET 11.1 (doc-1596): Simplifying ratios
- SkillsHEET 11.2 (doc-1597): Corresponding sides of congruent and similar triangles
- SkillsHEET 11.3 (doc-1598): Similar triangles
- SkillsHEET 11.4 (doc-1599): Using tests to prove similar triangles
- SkillsHEET 11.5 (doc-1600): Finding the scale factor

**11A Similar figures and scale factors****Interactivity**

- int-2403: Similarity (page 363)

**Tutorial**

- **WE4** int-2329: Learn to determine the scale factor between two similar figures (page 364)

**Digital docs**

- SkillsHEET 11.1 (doc-1596): Simplifying ratios (page 365)
- SkillsHEET 11.2 (doc-1597): Corresponding sides of congruent and similar triangles (page 365)
- SkillsHEET 11.3 (doc-1598): Similar triangles (page 365)
- SkillsHEET 11.4 (doc-1599): Using tests to prove similar triangles (page 365)

- SkillsHEET 11.5 (doc-1600): Finding the scale factor (page 365)
- WorkSHEET 11.1 (doc-1601): Apply your knowledge of scale factor to questions. (page 366)

**11B Solving problems using similar figures****Tutorial**

- **WE5** int-2330: Learn to determine the height of a tree using similar figures. (page 369)

**11C House plans****Tutorial**

- **WE9** int-2331: Learn to work with house plans. (page 373)

**Digital docs**

- WorkSHEET 11.2 (doc-1602): Apply your knowledge of similar figures to questions. (page 376)

**Chapter review**

- Test yourself Chapter 11 (doc-1603): Take the end-of-chapter test to test your progress. (page 381)

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